APPENDIX A.

LETTERS, AND PARTS OF LETTERS, PUBLISHED IN NATURE,

Between March 26 and June 18, 1874; here reproduced (with notes) as further elucidating the doctrine respecting physical axioms, set forth in the foregoing "Replies to Criticisms."

(from Nature, April 16, 1874.)

Absence from town has delayed what further remarks I have to make respecting the disputed origin of physical axioms.

The particular physical axiom in connection with which the general question was raised, was the Second Law of Motion. It stands in the Principia as follows:—

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

"If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force) if the body moved before, is added to or subducted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both."

As this, like each of the other Laws of Motion, is called an
axiom; as the paragraph appealed to it is simply an amplification, or re-statement in a more concrete form; as there are no facts named as bases of induction, not any justifying experiment; and as Newton proceeds forthwith to draw deductions; it was a legitimate inference that he regarded his truth as a priori. My statement to this effect was based on the contents of the Principia itself; and I think I was warranted in assuming that the nature of the Laws of Motion, as conceived by Newton, was to be thence inferred.

The passages quoted by the British Quarterly Reviewer from Newton’s correspondence, which were unknown to me, show that this was not Newton’s conception of them. Thus far then, my opponent has the best of the argument. Several qualifying considerations have to be set down, however.

(1) Clearly, the statements contained in the Principia do not convey Newton’s conception; otherwise there would have been no need for his explanations. The passages quoted prove that he wished to exclude these cardinal truths from the class of hypotheses which he said he did not make; and to do this he had to define them.

(2) By calling them “axioms,” and by yet describing them as principles “deduced from phenomena,” he makes it manifest that he gives the word “axiom” a sense widely unlike the sense in which it is usually accepted.

(3) Further, the quotations fail to warrant the statement that the Laws of Motion are proved true by the truth of the Principia. For if the fulfilment of astronomical predictions made in pursuance of the Principia, is held to be the evidence “on which they chiefly rest to this day,” then, until thus justified, they are unquestionably hypotheses. Yet Newton says they are not hypotheses.

Newton’s view may be found without seeking for it in his

* It is true that in Newton’s time, “axiom” had not the same rigorously defined meaning as now; but it suffices for my argument that, standing unproved as a basis for physical deductions, it bears just the same relation to them that a mathematical axiom does to mathematical deductions.
letters: it is contained in the *Principia* itself. The scholium to Corollary VI. begins thus:—

"Hitherto I have laid down such principles as have been received by mathematicians, and are confirmed by abundance of experiments. By the two first Laws and the two first Corollaries, Galileo discovered that the descent of bodies observed the duplicate ratio of the time, and that the motion of projectiles was in the curve of a parabola; experience agreeing with both," &c.

Now as this passage precedes the deductions constituting the *Principia*, it shows conclusively, in the first place, that Newton did not think "the whole of the *Principia* was the proof" of the Laws of Motion, though the Reviewer asserts that it is. Further, by the words I have italicised, Newton implicitly describes Galileo as having asserted these Laws of Motion, if not as gratuitous hypotheses (which he says they are not), then as *a priori* intuitions. For a proposition which is confirmed by experiment, and which is said to agree with experience, must have been entertained before the alleged verifications could be reached. And as before he made his experiments on falling bodies and projectiles, Galileo had no facts serving as an inductive basis for the Second Law of Motion, the law could not have been arrived at by induction.

Let me end what I have to say on this vexed question by adding a further reason to those I have already given, for saying that physical axioms cannot be established experimentally. The belief in their experimental establishment rests on the tacit assumption that experiments can be made, and conclusions drawn from them, without any truths being postulated. It is forgotten that there is a foundation of preconceptions without which the perceptions and inferences of the physicist cannot stand—preconceptions which are the products of simpler experiences than those yielded by consciously-made experiments. Passing over the many which do not immediately concern us, I will name only that which does,—the exact quantitative relation [of proportionality] between cause and effect. It is taken by the chemist as a truth needing no proof, that if two volumes of hydrogen unite with one volume of oxygen to form a certain
quantity of water, four volumes of hydrogen uniting with two volumes of oxygen will form double the quantity of water. If a cubic foot of ice at 32° is liquefied by the specified quantity of heat, it is taken to be unquestionable that three times the quantity of heat will liquefy three cubic feet. And similarly with mechanical forces, the unhesitating assumption is that if one unit of force acting in a given direction produces a certain result, two units will produce twice the result. Every process of measurement in a physical experiment takes this for granted; as we see in one of the simplest of them—the process of weighing. If a measured quantity of metal, gravitating towards the Earth, counterbalances a quantity of some other substance, the truth postulated in every act of weighing is, that any multiple of such weight will counterbalance an equi-multiple of such substance. That is to say, each unit of force is assumed to work its equivalent of effect in the direction in which it acts. Now this is nothing else than the assumption which the Second Law of Motion expresses in respect to effects of another kind. "If any force generates a motion, a double force will generate a double motion," &c., &c.; and when carried on to the composition of motions, the law is, similarly, the assertion that any other force, acting in any other direction, will similarly produce in that direction a proportionate motion. So that the law simply asserts the exact equivalence [or proportionality] of causes and effects of this particular class, while all physical experiments assume this exact equivalence [or proportionality] among causes and effects of all classes. Hence, the proposal to prove the Law of Motion experimentally, is i.e proposal to make a wider assumption for the purpose of justifying one of the narrower assumptions included in it.

Reduced to its briefest form, the argument is this: If definite quantitative relations [of proportionality] between causes and effects be assumed a priori, then, the Second Law of Motion is an immediate corollary. If there are not definite quantitative relations [of proportionality] between causes and effects, all the conclusions drawn from physical experiments are invalid. And
further, in the absence of this *a priori* assumption of equivalence, the quantified conclusion from any experiment may be denied, and any other quantification of the conclusion asserted.*

Herbert Spencer.

Entire misconstruction of the view expressed above, having been shown by a new assailant, who announced himself as also "A Senior Wrangler," Mr. James Collier wrote on my behalf an explanatory letter, published in *Nature* for May 21, from which the following passages are extracts:—

"The cue may be taken from an experience described in Mr. Spencer's *Principles of Psychology* (§ 468, note), where it is shown that when with one hand we pull the other, we have in the feeling of tension produced in the limb pulled, a measure of the reaction there is equivalent to the action of the other limb. Both terms of the relation of cause and effect are in this case present to consciousness as muscular tensions, which are our symbols of forces in general. While no motion is produced they are felt to be equal, so far as the sensations can serve to measure equality; and when excess of tension is felt in one arm, motion is experienced in the other. Here, as in the examples about to be given, the relation between cause and effect, though numerically indefinite, is definite in the respect that every additional increment of cause produces an additional increment of effect; and it is out of this and similar experiences that the idea of the relation of proportionality grows and becomes organic.

"A child, when biting its food, discovers that the harder he bites the deeper is the indentation; in other words, that the

* The above letter, written after absence at Easter had involved a week's delay, and written somewhat hurriedly to prevent the delay of a second week, was less carefully revised than it should have been. The words in square brackets, obviously implied by the reasoning, and specifically implied by the illustrations, were not in the letter as originally published.
more force applied, the greater the effect. If he tears an object with his teeth, he finds that the more he pulls the more the thing yields. Let him press against some thing solid, as his own person, or his clothes, or a lump of clay, and he sees that the part or object pressed yields little or much, according to the amount of the muscular strain. He can bend a stick the more completely the more force he applies. Any elastic object, as a piece of india-rubber or a catapult, can be stretched the farther the harder he pulls. If he tries to push a small body, there is little resistance and it is easy to move; but he finds that a big body presents greater resistance and is harder to move. The experience is precisely similar if he attempts to lift a big body and a little one; or if he raises a limb, with or without any object attached to it. He throws a stone: if it is light, little exertion propels it a considerable distance; if very heavy, great exertion only a short distance. So, also, if he jumps, a slight effort raises him to a short height, a greater effort to a greater height. Blowing with his mouth he sees that he can move small objects, or the surface of his morning's milk, gently or violently according as the blast is weak or strong. And it is the same with sounds: with a slight strain on the vocal organs he produces a murmur; with great strain he can raise a shout.

"The experiences these propositions record all implicate the same consciousness—the notion of proportionality between force applied and result produced; and it is out of this latent consciousness that the axiom of the perfect quantitative equivalence of the relations between cause and effect is evolved. To show how rigorous, how irreversible, this consciousness becomes, take a boy and suggest to him the following statements:—Can he not break a string he has, by pulling? tell him to double it, and then he will break it. He cannot bend or break a particular stick: let him make less effort and he will succeed. He is unable to raise a heavy weight: tell him he errs by using too much force. He can't push over a small chest: he will find it easier to upset a larger one. By blowing hard he cannot move a given object: if he blows lightly, he will move it. By great exertion
he cannot make himself audible at a distance: but he will make himself heard with less exertion at a greater distance. Tell him to do all or any of these, and of course he fails. The propositions are unthinkable and their unthinkableness shows that the consciousness which yields them is irreversible. These, then, are conceptions, properly so called, which have grown unconsciously out of the earliest experiences, beginning with those of the sucking infant, are perpetually confirmed by fresh experiences, and have at last become organized in the mental structure.

* * * * *

"Mr. Spencer's argument appears to be briefly this:—1. There are numberless experiences unconsciously acquired and unconsciously accumulated during the early life of the individual (in harmony with the acquisitions of all ancestral individuals) which yield the preconception, long antecedent anything like conscious physical experiments, that physical causes and effects vary together quantitatively. This is gained from all orders of physical experiences, and forms a universal preconception respecting them, which the physicist or other man of science brings with him to his experiments.

"2. Mr. Spencer showed in three cases—chemical, physical, and mechanical—that this preconception, so brought, was tacitly involved in the conception which the experimenter drew from the results of his experiments.

"3. Having indicated this universal preconception, and illustrated its presence in these special conceptions, Mr. Spencer goes on to say that it is involved also in the special conception of the relation between force and motion, as formulated in the 'Second Law of Motion.' He asserts that this is simply one case out of the numberless cases in which all these consciously-reasoned conclusions rest upon the unconsciously-formed conclusions that precede reasoning. Mr. Spencer alleges that as it has become impossible for a boy to think that by a smaller effort he can jump higher, and for a shopman to think that smaller weights will outbalance greater quantities, and for the
physicist to think that he will get increased effects from diminished causes, so it is impossible to think that 'alteration of motion' is not 'proportional to the motive force impressed.' And he maintains that this is, in fact, a latent implication of 'unconsciously-organized experiences just as much as those which the experimenter necessarily postulates.'

To meet further misinterpretations, a second letter was written by Mr. Collier and published in Nature for June 4. The following are passages from it:—

"Having but limited space, and assuming that the requisite qualifications would be made by unbiased readers, I passed over all those details of the child's experiences which would have been required in a full exposition. Of course I was aware that in the bending of a stick the visible effect does not increase in the same ratio as the force applied; and hardly needed the 'Senior Wrangler' to tell me that the resistance to a body moving through a fluid increases in a higher ratio than the velocity. It was taken for granted that he and those who think with him, would see that out of all these experiences, in some of which the causes and effects are simple, and in others of which they are complex, there grows the consciousness that the proportionality is the more distinct the simpler the antecedents and consequents. This is part of the preconception which the physicist brings with him and acts upon. Perhaps it is within the 'Senior Wrangler's' knowledge of physical explanation, that when the physicist finds a result not bearing that ratio to its assigned cause which the two were ascertained in other cases to have, he immediately assumes the presence of some perturbing cause or causes, which modify the ratio. There is, in fact, no physical determination made by any experimenter which does not assume, as an a priori necessity, that there cannot be a deviation from proportion without the presence of such additional cause."
“Returning to the general issue, perhaps the ‘Senior Wrangler’ will pay some respect to the judgment of one who was a Senior Wrangler too, and a great deal more—who was distinguished not only as a mathematician but as an astronomer, a physicist, and also as an inquirer into the methods of science: I mean Sir John Herschel. In his *Discourse on the Study of Natural Philosophy*, he says:—

‘When we would lay down general rules for finding and facilitating our search, among a great mass of assembled facts, for their common cause, we must have regard to the characters of that relation which we intend by cause and effect.’

‘Of these ‘characters’ he sets down the third and fourth in the following terms:—

‘Increase or diminution of the effect, with the increased or diminished intensity of the cause, in cases which admit of increase and diminution.’

‘Proportionality of the effect to its cause in all cases of direct unimpeled action.’

‘Observe that, in Sir J. Herschel’s view, these are ‘characters’ of the relation of cause and effect to be accepted as ‘general rules for guiding and facilitating our search’ among physical phenomena—truths that must be taken for granted before the search, not truths derived from the search. Clearly, the ‘proportionality of the effect to its cause in all cases of direct and unimpeled action’ is here taken as *à priori*. Sir J. Herschel would, therefore, have asserted, with Mr. Spencer, that the Second Law of Motion is *à priori*; since this is one of the cases of the ‘proportionality of the effect to its cause.’

‘And now let the ‘Senior Wrangler’ do what Sir J. Herschel has not done or thought of doing—prove the proportionality of cause and effect. Neither he, nor any other of Mr. Spencer’s opponents, has made the smallest attempt to deal with this main issue. Mr. Spencer alleges that this cognition of proportionality is *à priori*: not in the old sense, but in the sense that it grows out of experiences that precede reasoning. His opponents, following Prof. Tait in the assertion that Physics is a purely experimental science, containing, therefore, no *à priori*
truths, affirm that this cognition is à posteriori—a product of conscious induction. Let us hear what are the experiments. It is required to establish the truth that there is proportionality between causes and effects, by a process which nowhere assumes that if one unit of force produces a certain unit of effect, two units of such force will produce two units of such effect. Until the 'Senior Wrangler' has done this he has left Mr. Spencer's position untouched.
APPENDIX B.

FURTHER CORRESPONDENCE.

[After publication of the letters from which the foregoing are reproduced, there appeared in Nature certain rejoinders, containing misrepresentations even more extreme than those preceding them. There resulted a direct correspondence with two of the writers—Mr. Robert B. Hayward, of Harrow, and Mr. J. F. Moulton, my original assailant, the author of the article in the British Quarterly Review. This correspondence, in which I demanded from those gentlemen the justifications for their statements, formed part of this Appendix in its pamphlet form, as distributed among those who are competent to judge of the questions at issue. It is needless to give permanence to the replies and rejoinders. The character of Mr. Moulton's allegations, quite congruous with those I have exposed in the "Replies to Criticisms," may be inferred from one of the sentences closing my reply—"...wonderful to relate, my inductive proof that proportionality [of cause and effect] is taken for granted, he cites as my inductive proof of proportionality itself!" The result of the interchange of letters with Mr. Hayward, was to make it clear that "the thing I assert is not really disputed; and the thing disputed, I have nowhere asserted." While, however, the controversial part of the correspondence may fitly disappear, I retain an expository part embodied in the following letter to Mr. Hayward.]

38, Queen's Gardens, Bayswater, June 21st, 1874.

Sir,—Herewith I send you a copy of your letter with my interposed comments. I think those comments will make it clear to you that I have not committed myself to three different definitions of our consciousness of the Second Law of Motion.

As others may still feel a difficulty, such as you seem to have
felt, in understanding that which familiarity has made me regard as simple, I will endeavour, by a synthetic exposition, to make clear the way in which these later and more complex products of organized experiences stand related to earlier and simpler products. To make this exposition easier to follow, I will take first our Space-consciousness and the derived conceptions.

On the hypothesis of Evolution, the Space-consciousness results from organized motor, tactual, and visual experiences. In the Principles of Psychology, §§ 326—346, I have described in detail what I conceive to have been its genesis. Such Space-consciousness so generated, is one possessed in greater or less degree by all creatures of any intelligence; becoming wider, and more definite, according to the degree of mental evolution which converse with the environment has produced. How deeply registered the external relations have become in the internal structure, is shown by the facts that the decapitated frog pushes away with one or both legs the scalpel applied to the hind part of its body, and that the chick, as soon as it has recovered from the exhaustion of escaping from the egg, performs correctly-guided actions (accompanied by consciousness of distance and direction) in picking up grains. Ascending at once to such organized and inherited Space-consciousness as exists in the child, and which from moment to moment it is making more complete by its own experiences (aiding the development of its nervous system into the finished type of the adult, by the same exercises that similarly aid the development of its muscular system), we have to observe that, along with increasingly-definite ideas of distance and direction, it gains unawares certain more specific ideas of geometrical relations. Take one group of these. Every time it spreads open its fingers it sees an increase of the angles between them, going along with increase of the distances between the finger-tips. In opening wide apart its own legs, and in seeing others walk, it has continually before it the relation between increase or decrease of base in a triangle having equal sides, and increase or decrease
of the angle included by those sides. [The relation impressed on it being simply that of concomitant variation: I do not speak of any more definite relation, which, indeed, is unthinkable by the young.] It does not observe these facts in such way as to be conscious that it has observed them; but they are so impressed upon it as to establish a rigid association between certain mental states. Various of its activities disclose space-relations of this class more definitely. The drawing of a bow exhibits them in another way and with somewhat greater precision; and when, instead of the ends of a bow, capable of approaching one another, the points of attachment are fixed and the string elastic, the connexion between increasing length in the sides of an isosceles triangle and increasing acuteness of the included angle, is still more forced upon the attention; though it still does not rise into a conscious cognition. This is what I mean by an "unconsciously-formed preconception." When, in course of time, the child, growing into the boy, draws isosceles triangles on paper, and, among other things, draws isosceles triangles, the truth that, the base being the same, the angle at the apex becomes more acute as the sides lengthen, is still more definitely displayed to him; and when his attention is drawn to this relation he finds that he cannot think of it as being otherwise. If he imagines the lengths of the sides to change, he cannot exclude the consciousness of the correlative change in the angle; and presently, when his mental power is sufficiently developed, he perceives that if he continues to lengthen the sides in imagination, the lines approach parallelism as the angle approaches zero; yielding a conception of the relations of parallel lines. Here the consciousness has risen into the stage of definite conception. But, manifestly, the definite conception so reached is but a finishing of the preconceptions previously reached, and would have been impossible in their absence; and these unconsciously-formed preconceptions would similarly have been impossible in the absence of the still earlier consciousnesses of distance, direction, relative position, embodied in the consciousness of Space. The whole evolution
is one; the arrival at the distinct conception is the growing up to an ultimate definiteness and definiteness, and it can no more be reached without passing through the earlier stages of indefinite consciousness, than the adult bodily structure can be reached without passing through the structures of the embryo, the infant, and the child.*

Through a parallel evolution arises, first the vague consciousness of forces as exerted by self and surrounding things; presently, some discrimination in respect of their amounts as related to their effects; later, an association formed unawares

* Here, in explaining the genesis of special space-intuitions, I have singled out a group of experiences which, in *Nature*, May 28, Mr. Hayward had chosen as illustrating the absurdity of supposing that the scientific conception of proportionality could be reached as alleged. He said:—

"It is hardly a parody of Mr. Collier's remarks to say:—'A child discovers that the greater the angle between his legs the greater the distance between his feet, an experience which implies the notion of proportionality between the angle of a triangle and its opposite side;' a preconception, as it appears to me, with just as much a basis as that whose formation Mr. Collier illustrates, but one which, as I need hardly add, is soon corrected by a conscious study of geometry or by actual measurement."

I am indebted to Mr. Hayward for giving this instance, which conveniently serves two purposes. It serves to exemplify the connexion between the crude preconceptions unconsciously formed by earlier experiences, and the conceptions consciously evolved out of them by the help of later experiences, when the requisite powers of analysis and abstraction have been reached. And at the same time it serves to show the failure of my opponents to understand how, in the genesis of intelligence, the scientific conception of exact proportionality develops from the crude, vague, and inaccurate preconception. For while the notion of proportionality acquired by the child in Mr. Hayward's example, is not true, it is an approximation towards one which is true, and one which is reached when its more developed intelligence is brought critically to bear on the facts. Eventually it is discovered that the angle is not proportional to the subtending arc, but to the subtending arc; and this is discovered in the process of disentangling a simple relation from other relations which complicate and disguise it. Between the angle and the arc there is exact proportionality, for the reason that only one set of directly-connected space-relations are concerned: the distance of the subtending arc from the subtended angle, remains constant—there is no change in the relation between the increasing angle and the increasing arc; and therefore the two vary together in direct proportion. But it is otherwise with the subtend-
between greatness of quantity in the two, and between smallness of quantity in the two; later still, a tacit assumption of proportionality, though without a distinct consciousness that the assumption has been made; and, finally, a rising of this assumption into definite recognition, as a truth necessarily holding where the forces are simple. Throughout its life every creature has, within the actions of its moving parts, forces and motions conforming to the Laws of Motion. If it has a nervous system, the differences among the muscular tension and the movements initiated, register themselves in a vague way in that nervous system. As the nervous system develops, along with more developed limbs, there are at once more numerous different experiences of momentum generated, of connected actions and reactions (as when an animal tears the food which it holds with its paws); and, at the same time, there are, in its more developed nervous system, increased powers of appreciating and registering these differences. All the resulting connexions in consciousness, though unknowingly formed and unknowingly entertained, are

The parts of this stand in different relations of distance from the subtended angle; and as the line is lengthened, each added part differs from the preceding part in its distance from the angle. That is to say, one set of simple directly-connected geometrical relations, is here involv'd with another set; and the relation between the side and the angle is such that the law of relative increase involves the co-operation of two sets of factors. Now the distinguishing the true proportionality (between the angle and the arc) from the relation which simulates proportionality (between the angle and the side) is just that process of dual development of exact conceptions, which I assert to be the finishing step of all the preceding development; and I to be impossible in its absence. And the truth to which my assailants shut their eyes is that, just as among these conceptions of space-relations the conception of exact proportionality can be reached only by evolution from the crude notion of proportionality formed before reasoning begins; so, among the force-relations, the conception of proportionality finally reached, when simple cause and their effects are disentangled by analytical intelligence, can be reached only by evolution of the crude notion of proportionality, established as a preconception by early experiences which reinforce ancestral experiences.
ever present as guides to action, witness the proportion between the effort an animal makes and the distance it means to spring; or witness the delicate adjustments of muscular strains to changes of motion, made by a swallow catching flies or a hawk swooping on its quarry. Manifestly, then, these experiences, organized during the earlier stages of mental evolution, form a body of consciousness, not formulated into cognitions, not present even as preconceptions, but nevertheless present as a mass of associations in which the truths of relation between force and motion are potentially present. On ascending to human beings of the uncultured sort, we reach a stage at which some nascent generalization of these experiences occurs. The savage has not expressed to himself the truth that if he wants to propel his spear further he must use more force; nor does the rustic put into a distinct thought the truth that to raise double the weight he must put forth twice the effort; but in each there is a tacit assumption to this effect, as becomes manifest on calling it in question. So that, in respect of these and other simple mechanical actions, there exist unconsciously-formed preconceptions. And just as the geometrical truths presented in a rude way by the relations among surrounding objects, are not overtly recognized until there is some familiarity with straight lines, and diagrams made of them; so, until linear measures, long used, have led to the equal-armed lever, or scales, and thus to the notion of equal units of force, this mechanical preconception cannot rise into definiteness. Nor after it has risen into definiteness does it for a long time reach the form of a consciously-held cognition; for neither the village butcher nor the more cultivated druggist in the town, recognizes the general abstract truth that, when uninterfered with, equi-multiples of causes and their effects are necessarily connected. But now observe that this truth, acted upon with more or less distinct consciousness of it by the man of science and perfected by him through analysis and abstraction, is thus perfected only as the last step in its evolution. This definite cognition is but the finished form of a consciousness long in preparation—a con-
sciousness the body of which is present in the brute, takes some shape in the primitive man, reaches greater definiteness in the semi-civilized, becomes afterwards an assumption distinct though not formulated, and takes its final development only as it rises into a consciously-accepted axiom. Just as there is a continuous evolution of the nervous system, so is there a continuous evolution of the consciousness accompanying its action; just as the one grows in volume, complexity, and definiteness, so does the other; and just as necessary as the earlier stages are to the later in the one case, are they in the other. To suppose that the finished conceptions of science can exist without the unfinished common knowledge which precedes them, or this without still earlier mental acquisitions, is the same thing as to suppose that we can have the correct judgments of the adult without passing through the crude judgments of the youth, the narrow, incoherent ones of the child, and the vague feeble ones of the infant. So far is it from being true that the view of physical axioms held by me, is one which bases cognitions on some other source than experience, it asserts experience to be the only possible source of these, as of other conceptions; but it asserts, further, that not simply is the consciously-acquired experience of the present needful, but that for the very possibility of gaining it we are indebted to the accumulation of all past experiences. Not I, but my antagonist, are really chargeable with accepting the ancient a priori view; since, without any explanation of them or justification of them, they posit as unquestionable the assumptions underlying every experiment and the conclusion drawn from it. The belief in physical causation, assumed from moment to moment as necessary in every experiment and in all reasoning from it, is a belief which, if not justified by the hypothesis above set forth, is tacitly asserted as an a priori belief. Contrariwise, my own position is one which affiates all such beliefs upon experiences acquired during the whole past; which alleges those experiences as the only warrant for them; which asserts that during the converse between the mind and its environment, necessary
connexions in Thought, such as those concerning Space, have resulted from infinite experiences of corresponding necessary connexions in Things; and that, similarly, out of perpetual converse with the Forces manifested to us in Space, there has been a progressive establishment of internal relations answering to external relations, in such wise that there finally emerge as physical axioms, certain necessities of Thought which answer to necessities in Things.

I need scarcely say that I have taken the trouble of making my comments on your letter, and of writing this further exposition, with a view to their ulterior use.

I am, &c.,

HERBERT SPENCER.
APPENDIX C.

SUMMARY OF RESULTS.

Those who deny a general doctrine enunciated by Mayer as the basis of his reasonings, habitually assumed by Faraday as a guiding principle in drawing his conclusions, distinctly held by Helmholtz, and tacitly implied by Sir John Herschel—those, I say, who deny this general doctrine and even deride it, should be prepared with clear and strong reasons for doing this. Having been attacked, not in the most temperate manner, for enunciating this doctrine and its necessary implications in a specific form, I have demanded such reasons. Observe the responses to the demand.

1. The British Quarterly Review quoted for my instruction the dictum of Professor Tait, that "Natural Philosophy is an experimental, and not an intuitive science. No a priori reasoning can conduct us demonstratively to a single physical truth." Thereupon I inquired what Professor Tait meant "by speaking of physical axioms; and by saying that the cultured are enabled to see at once their necessary truth?" No reply.

2. Instead of an answer to the question, how this intuition of necessity can be
alleged by Professor Tait consistently with his other doctrine, the Reviewer quotes, as though it disposed of my question, Professor Tait's statement that "as the properties of matter might have been such as to render a totally different set of laws axiomatic, these laws [of motion] must be considered as resting on convictions drawn from observation and experiment, and not on intuitive perception." Whereupon I inquired how Professor Tait knows that "the properties of matter might have been" other than they are. I asked how it happened that his intuition concerning things as they are not, is so certain that, by inference from it, he discredits our intuitions concerning things as they are.

\[\text{No reply: Professor Tait told,} \]
\[\text{à propos of my question, a story} \]
\[\text{of which no one could discover the} \]
\[\text{application; but,} \]
\[\text{otherwise, declined to answer. Nor was} \]
\[\text{any answer given by his disciple.} \]

3. Further, I asked how it happened that Professor Tait accepted as laws for Physics, Newton's Laws of Motion; which were illustrated but not proved by Newton, and of which no proofs are supplied by Professor Tait, in the Treatise on Natural Philosophy. I went on to examine what conceivable \( \text{à posteriori} \) warrant there could be if there
was no warrant \textit{a priori}; and I pointed out that neither from terrestrial nor from celestial phenomena could the First Law of Motion be deduced without a \textit{petitio principii}.

4. To ... assertion that Newton gave no proof of the \textit{Laws} of Motion, the Reviewer joined that “the whole of the \textit{Principia} was the proof.” On which my comment was that Newton called them “axioms,” and that axioms are not commonly supposed to be proved by deduction from them.

No reply: the Reviewer characterized my reasoning as “utterly erroneous” (therein differing entirely from two eminent authorities who read it in proof); but beyond so characterizing it he said nothing.

The Reviewer quotes from one of Newton’s letters a passage showing that though he called the Law of Motion “axioms,” he regarded them as principles “made general by induction;” and that therefore he
5. In rejoinder, I pointed out that whatever conception Newton may have had of these "axioms," he explicitly and distinctly excluded them from the class of "hypotheses." Hence I inferred that he did not regard the whole of the *Principia* as the proof, which the Reviewer says it is; since an assumption made at the outset, to be afterwards justified by the results of assuming it, is an "hypothesis"..............

No reply.

6. Authority aside, I examined on its merits the assertion that the Laws of Motion are, or can be, proved true by the ascertained truth of astronomical predictions; and showed that the process of verification itself assumed those Laws......

No reply.

7. To make still clearer the fact that ultimate physical truths are, and must be, accepted as *à priori*, I pointed out that in every experiment the physicist tacitly assumes a relation between cause and effect, such that, if one unit of cause produces its unit of effect, two units of the cause will produce two units of the effect; and I argued that this general assumption included the special assumption asserted in the Second Law of Motion..............

No reply: that is to say, no endeavour to show the untruth of this statement, but a
quibble based on my omission of the word "proportionality" in places where it was implied, though not stated.

8. Attention was drawn to a passage from Sir John Herschel's *Discourse on the Study of Natural Philosophy*, in which the "proportionality of the effect to its cause in all cases of direct unimpeded action" is included by him among "the characters of that relation which we intend by cause and effect;" and in which this assumption of proportionality is set down as one preceding physical explanation, and not as one to be established by it ............................ No reply.

9. Lastly, a challenge to prove this proportionality. "It is required to establish the truth that there is proportionality between causes and effects, by a process which nowhere assumes that if one unit of force produces a certain unit of effect, two units of such force will produce two units of such effect."................................ No reply.

Thus on all these essential points my three mathematical opponents allow judge .......................... to go against them by default. The attention of readers has been drawn off from the main issues by the discussion of side issues. Fundamental questions have been evaded, and new questions of subordinate kinds raised.

What is the implication? One who is able to reach and to carry the central position of his antagonist, does not spend
his strength on small outposts. If he declines to assault the stronghold, it must be because he sees it to be impregnable.

The trouble I have taken to meet criticisms and dissipate misapprehensions, I have taken because the attack made on the special doctrine defended, is part of an attack on the ultimate doctrine underlying the deductive part of *First Principles*—the doctrine that the quantity of existence is unchangeable. I agree with Sir W. Hamilton that our consciousness of the necessity of causation, results from the impossibility of conceiving the totality of Being to increase or decrease. The proportionality of cause and effect is an implication of it. The assertion that some quantity of cause has appeared without effect, or some quantity of effect has arisen without cause. I have asserted the *à priori* character of the Second Law of Motion, under the abstract form in which it is expressed, simply because this, too, is an implication, somewhat more remote, of the same ultimate truth. And my sole reason for insisting on the validity of these intuitions, is that, on the hypothesis of Evolution, absolute uniformities in things have produced absolute uniformities in thoughts; and that necessary thoughts represent infinitely-larger accumulations of experiences than the observations, experiments, and reasonings of any single life.